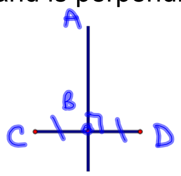
5.2 Use Perpendicular Bisectors

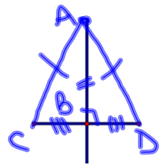
5.3 Use Angle Bisectors of Triangles

perpendicular bisector—is a line, segment, plane, or ray that passes through the midpoint of a segment and is perpendicular to it.

$\overline{AB}$ is perp. bis. of $\overline{CD}$



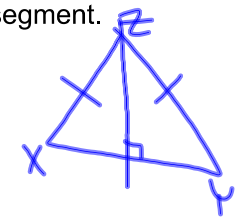
Thm 5.2—Perpendicular Bisector Theorem—In a plane, any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.



Given: $\overline{AB}$ perp. bis. of $\overline{CD}$
Prove: $\overline{AC} \cong \overline{AD}$

S	R
① ~	① Given
② $\overline{CB} \cong \overline{BD}$; $\angle ABC$ + $\angle ABD$ are Rt $\angle$s	② def of ⊥ Bis.
③ $\angle ABC \cong \angle ABD$	③ All Rt $\angle$s are $\cong$
④ $\overline{AB} \cong \overline{AB}$	④ Reflexive
⑤ $\triangle ACB \cong \triangle ADB$	⑤ SAS
⑥ $\overline{AC} \cong \overline{AD}$	⑥ CPCTC

Thm 5.3—Converse of the Perpendicular Bisector Theorem—In a plane, any point equidistant from the endpoints of the segment lies on the perpendicular bisector of the segment.

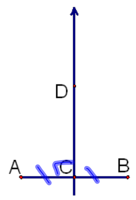


Examples:

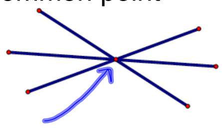
1. $\overline{CD}$ is a ⊥ bisector of $\overline{AB}$.
   m∠DCA = 2x. Solve for x. 45
   2x = 90

   AC = 3y + 2, BC = 14. Solve for y. 4
   3y + 2 = 14

   AD = 4z, BD = 20. Solve for z. 5
   4z = 20

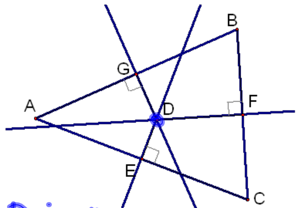


Concurrent lines—three or more lines that intersect at a common point

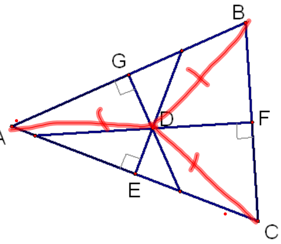


Point of concurrency—the point of intersection

Circumcenter—the point of concurrency of the perpendicular bisectors of a triangle.

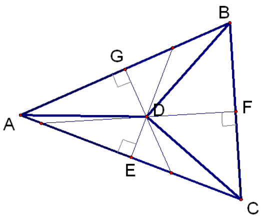


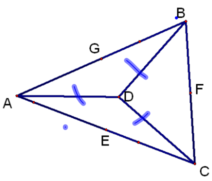
D is the circumcenter



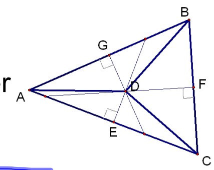
Thm 5.4 Circumcenter theorem—The circumcenter of a triangle is equidistant from the vertices of the triangle.

Conclusion: $\overline{DA} \cong \overline{DC} \cong \overline{DB}$





Proof:
Given: D is the circumcenter
Prove: $\overline{AD} \cong \overline{CD} \cong \overline{BD}$



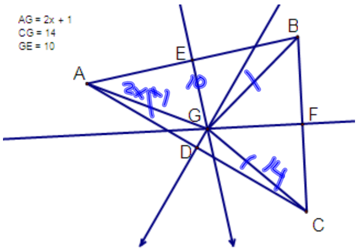
S	R
① ~	① Given
② $\overline{DE}$ ⊥ bis. $\overline{AC}$ $\overline{DG}$ ⊥ bis $\overline{AB}$ $\overline{DF}$ ⊥ bis $\overline{BC}$	② def of circumcenter
③ $\overline{AD} \cong \overline{DB}$ $\overline{DB} \cong \overline{CD}$ $\overline{AD} \cong \overline{CD}$	③ ⊥ Bis. thm.
④ $\overline{AD} \cong \overline{CD} \cong \overline{DB}$	④ Subst.

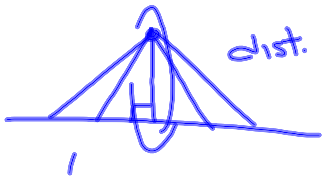
Ex 2. G is the circumcenter. x =

$2x+1=14$

$2x=13$

$x=6.5$





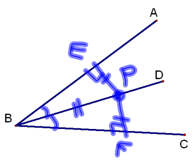
dist. b/w pt + line

5.3 Angle Bisectors

Thm 5.5—Angle Bisector Theorem--Any point on the angle bisector is equidistant from the sides of the angle.

Thm 5.6—Converse of the Angle Bisector Theorem—If a point is on the interior of an angle and it is equidistant from the sides of the angle, then it lies on the angle bisector.

Conclusion:__________________________



G: $\overrightarrow{BD}$ bis. ∠ABC

C: $\overline{PE} \cong \overline{PF}$

Ex 3. $\overrightarrow{BD}$ bisects ∠ABC. Solve for x.________

m∠ABC = 15x

m∠ABD = 10x – 8

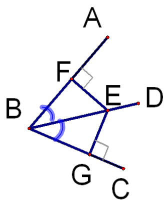
$15x = 2(10x-8)$

$3.2 = x$

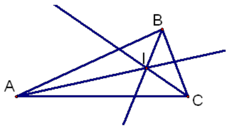
EF = y + 8

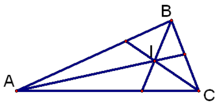
EG = 3y – 6 $= y+8$

y = 7



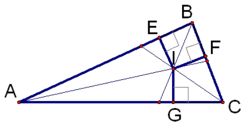
Incenter—The point of concurrency of the angle bisectors of a triangle.

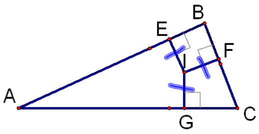




Thm 5.7 The Incenter Theorem—The incenter of a triangle is equidistant from each side of the triangle.

Conclusion: $\overline{IE} \cong \overline{IG} \cong \overline{IF}$



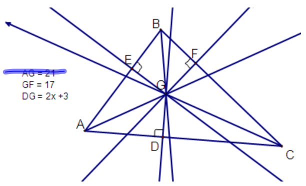


Ex 4. G is the incenter. x = ______.

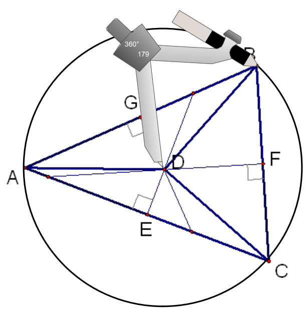
$2x+3=17$

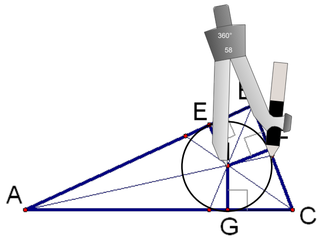
$2x=14$

$x=7$



Circles can be drawn around these centers.





HW
p307 #s 11-17
p309 quiz #s 1-3
p313-314 #s 3-8, 12-17, 24, 26